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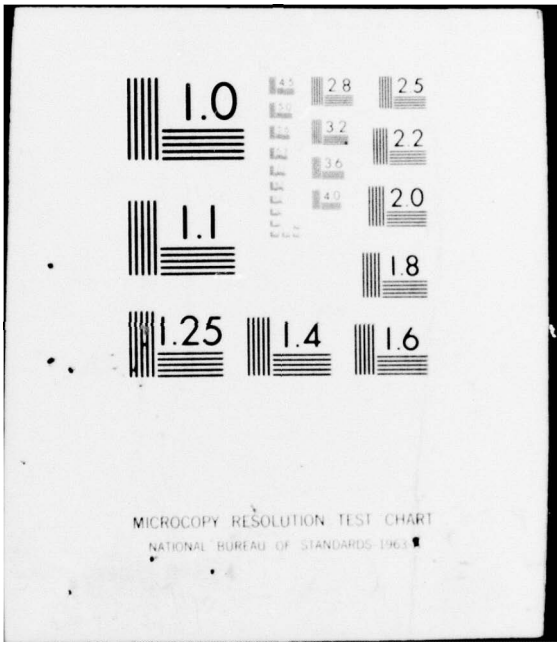
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by
ROLF FÄRE

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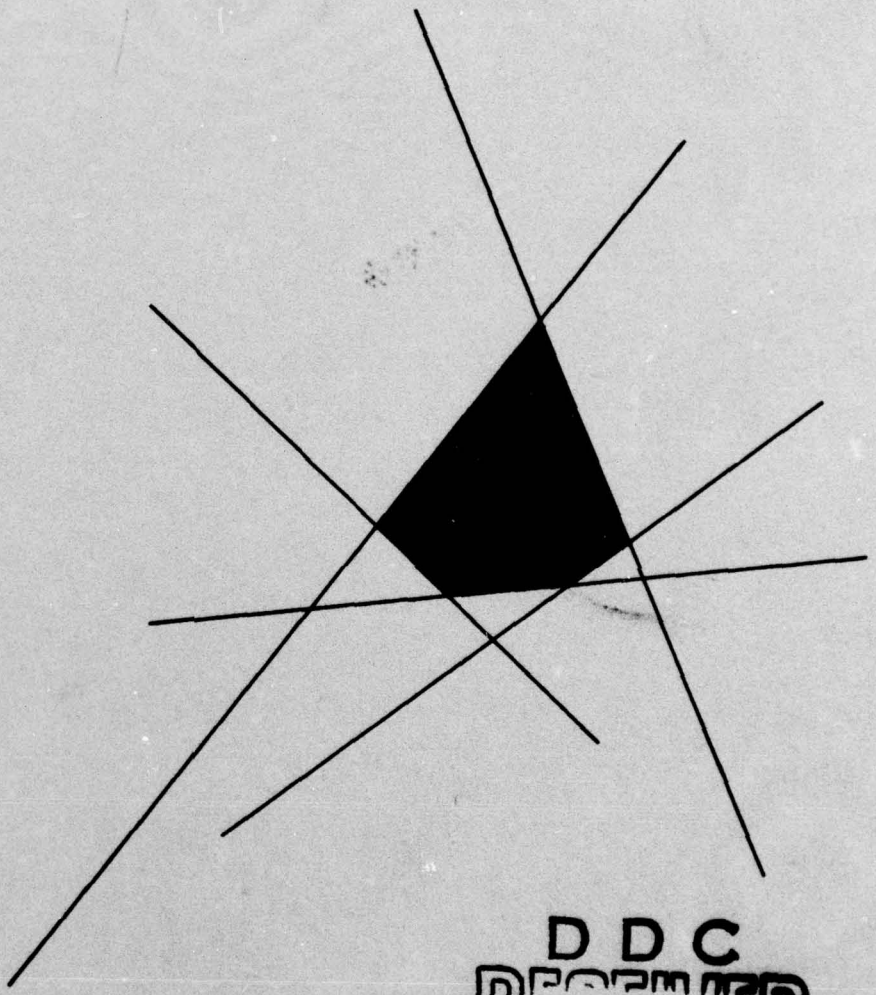

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by

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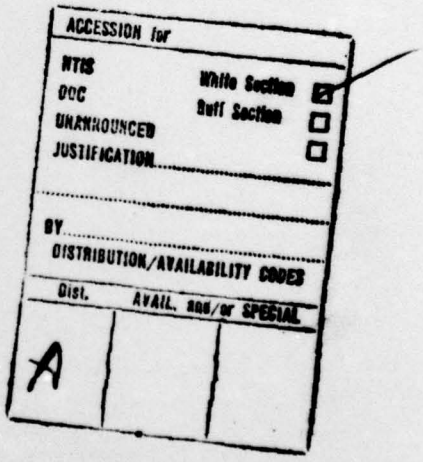

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ABSTRACT

A book by Dr. Hasenkamp on the theory and econometrics of multiple-output production functions is reviewed.

MULTIPLE-OUTPUT PRODUCTION FUNCTION, A REVIEW

by

Rolf Färe
Lund, Sweden

In a revision of his Ph.D. dissertation, entitled, "Specification and Estimation of Multiple-Output Production Functions," [4], published by Springer-Verlag as No. 120 of the Lecture Notes in Economics and Mathematical Systems, Dr. Georg Hasenkamp discusses theory and estimations of multiple-output production functions. The book of 151 + VI pages is divided into five chapters of which the first is a short introductory one. Chapters 2 and 3 deals with the theory of production while 4 and 5 are devoted to econometrics for the same.

Let me first consider the econometric results in Chapter 5 of which some are published elsewhere (see [5]). Dr. Hasenkamp has provided a thorough econometric analysis in the traditional way of econometrics. However this approach suffers from the failure to consider structural concepts such as essentiality and congestion which can serve as guidelines in econometric studies. For example, in his study of the U.S. railroads using the CES (Constant Elasticity of Substitution) input function, estimates are made (see [4, p. 94]) of the elasticity of substitution to be less than one, implying that, if any of the three inputs equals zero, only zero output is possible, i.e., each factor of production is essential by itself for both outputs. This is a reasonable result. On the other hand estimates are made with the elasticity of substitution larger than one (see [4, p. 116]), implying that no proper subset of factors of production is essential to any output; which appears unreasonable in this case.

To give another example, following tradition all of Dr. Hasenkamp's

input functions obey strong disposability of inputs, i.e., they are all nondecreasing over the whole input space. In such production models there can be no congestion (see [3]), i.e., for example if capital services are kept constant while labor and fuel are increased indefinitely in the railroad example, output (freight and passenger services) will never decrease. For such reasons, concepts like essentiality, congestion and others (see [6]) can play important roles in econometric studies.

Dr. Hasenkamp uses nonlinear estimation procedures. This raises the question of how to find "good" starting points. Clearly, he knows how, but it is not apparent to the reviewer how such starting points are determined. A more complete discussion on this problem is worth publication.

In Chapter 2, the author proves propositions such as Shephard's lemma, discusses almost-homogeneity, separability and subaggregation of inputs and outputs. Although the results are known, he gives a good treatment of these basic production theoretical notions.

The multiple-output production function $F : \mathbb{R}_+^{m+n} \to \mathbb{R}_+$, is defined on page 5. A combination (y,x) of outputs $y \in \mathbb{R}_+^m$ and inputs $x \in \mathbb{R}_+^n$ for which

$$F(y,x) = \alpha \tag{2.2}$$

is called an efficient output-input combination. Assuming efficiency in its usual meaning, i.e., (y,x) is efficient if (y,x) is a possible output-input combination and if $y^1 \geq y$ or $x \leq x^1$ ($\geq$ means $\geq$ but $\neq$), imply that (y^1,x^1) is not possible, then (2.2) is not reasonable. To see this, choose a specific form of (2.80), [4, p. 37], namely, $F(y,x) = \max\{y_1,y_2\} - x = 0$, $(x,y_1,y_2 \in \mathbb{R}_+)$. In this case, the output-input combination $(y_1,y_2,x) = (1,2,2)$ is clearly not efficient. A possible interpretation of the function $F(y,x)$ is that it is a joint production function along the lines of Shephard [6]. There,

$F(y,x)$ is a joint production function if for given $\bar{x}$, $F(y,\bar{x}) = 0$ if and only if y belongs to the isoquant of the output set and for given $\bar{y}$, $F(\bar{y},x) = 0$ if and only if x belongs to the isoquant of the input set. Shephard, [6], gave necessary conditions for the joint production function to exist and later, Bol and Moeshlin, [1], also gave sufficient conditions. These conditions are severe. Another possibility is to interpret $F(y,x)$ as an efficiency function along the lines of [2].

In Chapter 3, Dr. Hasenkamp gives various specific forms of the multiple-output production function and clearly states the different properties of the same. For reasons pointed out earlier one would also have liked to have had examples of functions with weak disposability of inputs and outputs. The relaxation of strong disposability can help to improve econometric studies (see [3]).

This book by Dr. Hasenkamp is interesting and more work of this kind is hoped for.

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